of larger reactor and that of small reactor in two parallel reactor scheme
k = rate constant combined with residence time, $k'\tau_o$
k', k_o = rate constant and preexponential factor
k_c', k_h' = rate constant corresponding to T_c and T_h
k_1, k_2 = $k_c'\tau_o$ and $k_h'T_o$, respectively
r = inverse of velocity normalized with respect to v_o, v_o/v
r_h = duration of normalized step-up period for periodic flow operation, inverse of normalized velocity during step-up period
r_l = duration of normalized step-down period for periodic flow operation, inverse of normalized velocity during step-down period
R = V_l/V_o
t = time normalized with respect to τ_o
t_o, t_h, t_l = residence time in two parallel reactor scheme; V_o/F_o, V_h/F_h *and* V_l/F_l
T, T_h, T_c = design operating temperature, feed temperature during step-down period and during step-up period
v_o, v_h, v_l = design feed velocity, velocity during step-up period and during step-down period
V_o, V_h, V_l = volume of reference reactor, that of larger reactor and that of smaller reactor in two parallel reactor scheme
x = normalized concentration of A, A/A_o
y = average value of $x\ (1, t)$
y_t = y for TPF operation
z = distance normalized with respect to reactor length
τ_h = duration of step-up period, residence time corresponding to v_h
τ_l = duration of step-down period, residence time corresponding to v_l
τ_o = residence time corresponding to v_o

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Heat Transfer in Plane Couette Flow with Additional Pressure Gradient Including Entrance Effects for a Non-Newtonian Fluid

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Analytical solutions are presented for laminar plane Couette flow of a power law non-Newtonian fluid. Effects of an additional negative pressure gradient were included in solving for the velocity distribution. The contributions due to the entrance heating region and the viscous dissipation effects were included in arriving at the temperature distribution and also the variation of the local Nusselt number at the stationary plate.

Heat transfer in plane Couette flow has been given due attention because of some important applications as pointed out by Bruin (1972). One example of such applications is the study of thermal pasteurization processing of liquid foods. As suggested by Bruin (1972), the death rate of microorganisms can be considered as a first-order chemical reaction, where the reaction constant is strongly temperature dependent. Another example is the extrusion of the paper pulp prior to rolling on the paper machines. Liquid foods or paper pulp are, in general, non-Newtonian, and therefore an analysis considering this fact would be more accurate than those found in the published work by Bruin (1972), Sestak and Rieger (1969), El-Ariny and Aziz (1976), and Hudson and Bankoff (1965). Tien (1961) considered the case of simple Couette flow of a non-Newtonian power law fluid; however, his solution is good only for fully developed velocity and temperature profiles. Tien (1962) also extended the Graetz-Nusselt problem to non-Newtonian fluids, and Suckow et al. (1971) improved on it. However, these two solutions were for the case of two stationary plates, and it may not be suitable to compare them with the present work.

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The present work considers the case of laminar plane Couette flow, where the upper plate moves in its own plane with a constant velocity. This work also takes into account the effect of an additional negative pressure gradient on the flow field. In solving for the temperature distribution, the entrance heating region is also covered.

ANALYSIS

The physical model is shown in Figure 1, where the plates temperatures and the fluid inlet temperature are taken constant but different.

The shear stress in the fluid is assumed to follow the power law model, namely

$$\tau_{xy} = \kappa\,(\partial u/\partial y)\,|\partial u/\partial y|^{n-1} \tag{1}$$

The basic conservation equations pertinent to this model in nondimensional form are:

Momentum

$$\frac{d}{dy^*}\,[(du^*/dy^*)\,|du^*/dy^*|^{n-1}] = -\,Re\,(-\,dp^*/dx^*) \tag{2}$$

Energy

$$Pe\cdot u^*\,(\partial\theta/\partial x^*) = (\partial\theta^2/\partial y^{*2}) + (Pe\cdot Ec/Re)\,|du^*/dy^*|^{n+1} \tag{3}$$

Boundary conditions

$$u^*(0) = 0, \quad u^*(1) = 1 \tag{4a}$$

$$\theta(x^*, 0) = 0, \quad \theta(x^*, 1) = 1, \quad \theta(0, y^*) = \theta_i \tag{4b}$$

where

$$x^* = x/a, \quad y^* = y/a, \quad u^* = u/U,$$
$$p^* = p/\rho U^2, \quad \theta = (T - T_0)/(T_1 - T_0)$$

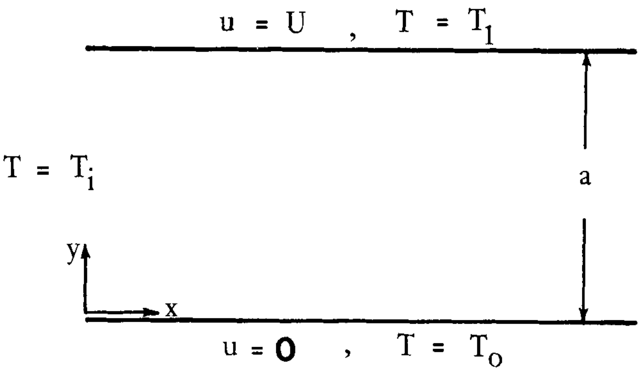

Fig. 1. The physical model.

Velocity distribution

The solution of (2) subject to (4*a*) is

$$u^* = \frac{1 - [1 - B\,y^*]^{(n+1)/n}}{1 - [1 - B]^{(n+1)/n}} \tag{5}$$

where *B* is given by

$$\frac{B^{n+1}\,[(n+1)/n]^n}{[1 - \{1 - B\}^{(n+1)/n}]^n} = Re\,(-\,dp^*/x^*) \tag{6}$$

Equation (6) is shown plotted in Figure 2, and Figure 3 shows the velocity distribution for various values of *n* and *B*.

Temperature distribution

Let

$$\theta(x^*, y^*) = \theta_1(x^*, y^*) + \theta_2(y^*) \tag{7}$$

such that

$$d^2\theta_2/dy^{*2} = -\,(Pe\cdot Ec/Re)\,|du^*/dy^*|^{n+1} \tag{8}$$

with

$$\theta_2(0) = 0 \quad \text{and} \quad \theta_2(1) = 1 \tag{9}$$

Solution of (8) subject to (9) gives

$$\theta_2 = y^* + D\,[[1 - \{1 - B\,y^*\}^{(3n+1)/n}] - y^*\,[1 - \{1 - B\}^{(3n+1)/n}]] \tag{10}$$

where

$$D = n^2\,N/B^2(2n+1)(3n+1)$$

and

$$N = (Pe\cdot Ec/Re)\,[nB(n+1)/[1 - \{1 - B\}^{(n+1)/n}]]$$

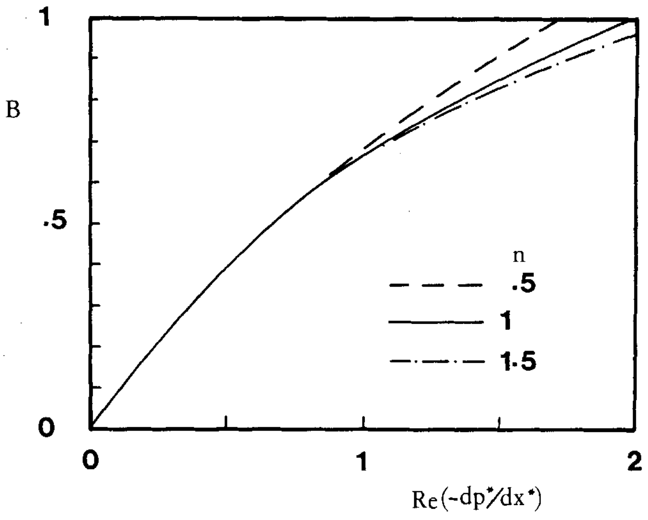

Fig. 2. Solution of equation (6).

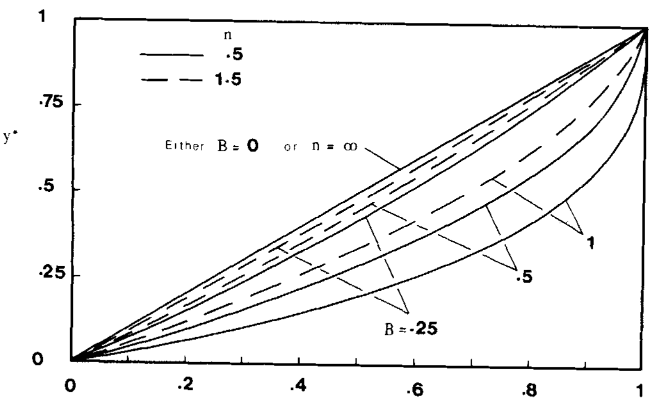

Fig. 3. Velocity distribution between the two plates.

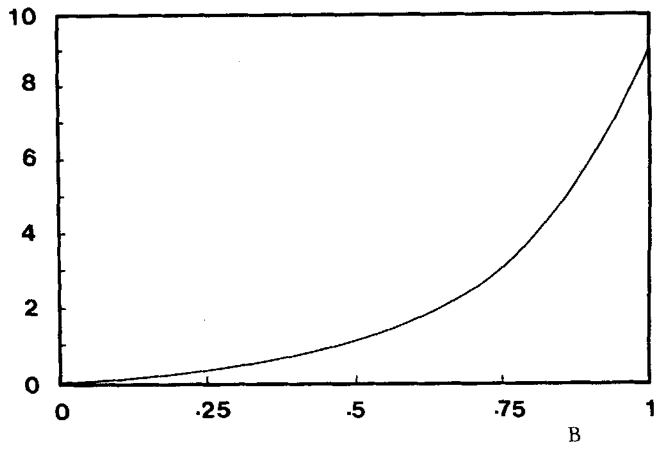

Fig. 4. Percent maximum deviation in velocity, for $0.25 < n < 1.5$.

This leaves out $\theta_1(x^*, y^*)$ which satisfies

$$Pe \cdot u^* \, \partial\theta_1/\partial x^* = \partial^2\theta_1/\partial y^{*2} \tag{11}$$

with

$$\theta_1(x^*, 0) = 0, \quad \theta_1(x^*, 1) = 0, \quad \text{and} \quad \theta_1(0, y^*) = \theta_i - \theta_2(y^*) \tag{12}$$

In order to solve for θ_1, one makes the following approximation in Equation (11):

$$u^* \simeq (1 - Z^2)/(1 - H^2) \tag{13}$$

where

$$Z = 1 - (B/n)\, y^*$$

and

$$H = 1 - (B/n)$$

In addition, if one writes

$$X = x^* \, B^2/4n^2 \, Pe(1 - H^2)$$

Equation (11) becomes

$$(1/4 - 1/4 Z^2)\,\partial\theta_1/\partial X = \partial^2\theta_1/\partial Z^2 \tag{14}$$

with

$$\theta_1(X, 1) = 0, \quad \theta_1(X, H) = 0, \quad \text{and} \quad \theta_1(0, Z) = \theta_i - \theta_2(Z) \tag{15}$$

Solution of (14) subject to (15) gives (see Abramowitz and Stegun, 1964)

$$\theta_1(X, Z) = \sum_{m=0}^{\infty} C_m \exp(-\lambda_m{}^2 X - 1/4\lambda_m Z^2) \, V(\lambda_m, Z) \tag{16}$$

where

$$V(\lambda_m, Z) = M\left(\frac{1}{4} - \frac{\lambda_m}{8}, \frac{1}{2}, \frac{\lambda_m}{2} Z^2\right) \cdot M\left(\frac{3}{4} - \frac{\lambda_m}{8}, \frac{3}{2}, \frac{\lambda_m}{2}\right) - Z \cdot M\left(\frac{1}{4} - \frac{\lambda_m}{8}, \frac{1}{2}, \frac{\lambda_m}{2}\right) \cdot M\left(\frac{3}{4} - \frac{\lambda_m}{8}, \frac{3}{2}, \frac{\lambda_m}{2} Z^2\right) \tag{17}$$

and

$$C_m = I_1/I_2 \tag{18}$$

where

$$I_1 = \int_1^H [\theta_i - \theta_2(Z)] \exp(-1/4\lambda_m Z^2) \, V \, dZ$$

$$I_2 = \int_1^H \exp(-1/2\lambda_m Z^2) \, V^2 \, dZ$$

λ_m is the eigenvalue, and its values are the roots of the characteristic equation

$$H \cdot M\left(\frac{1}{4} - \frac{\lambda}{8}, \frac{1}{2}, \frac{\lambda}{2}\right) \cdot M\left(\frac{3}{4} - \frac{\lambda}{8}, \frac{3}{2}, \frac{\lambda}{2} H^2\right) - M\left(\frac{1}{4} - \frac{\lambda}{8}, \frac{1}{2}, \frac{\lambda}{2} H^2\right) \cdot M\left(\frac{3}{4} - \frac{\lambda}{8}, \frac{3}{2}, \frac{\lambda}{2}\right) = 0 \tag{19}$$

The local heat transfer coefficient can be expressed in terms of the Nusselt number at the stationary plate as

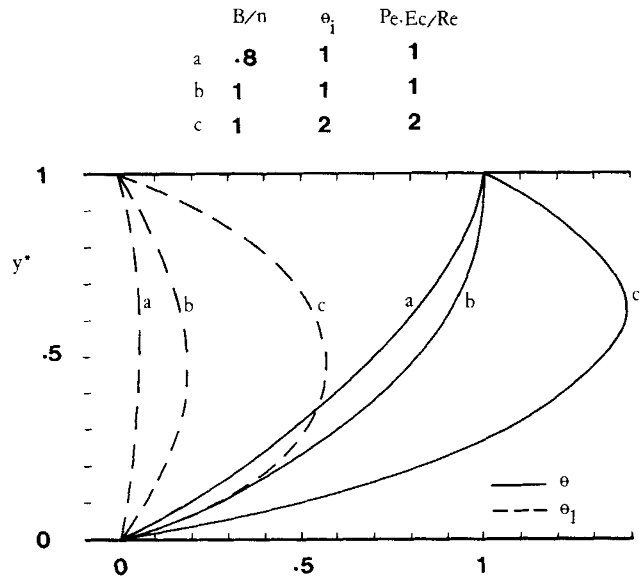

Fig. 5. Temperature profile between the plates at $X = 0.02$ for $n = 0.25$.

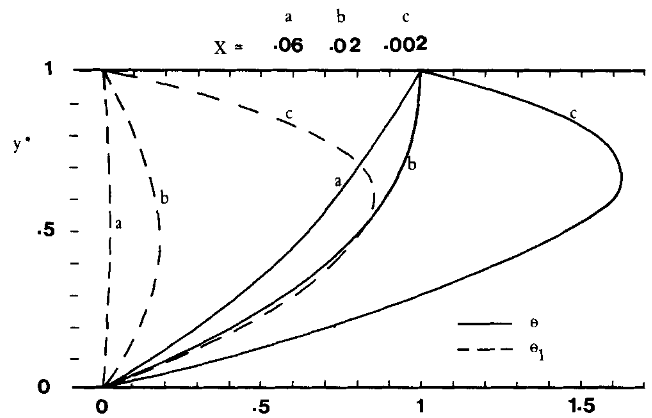

Fig. 6. Temperature profile between the plates for $n = 0.25$, $\Theta_i = 1$, and $(Pe.Ec/Re) = 1$.

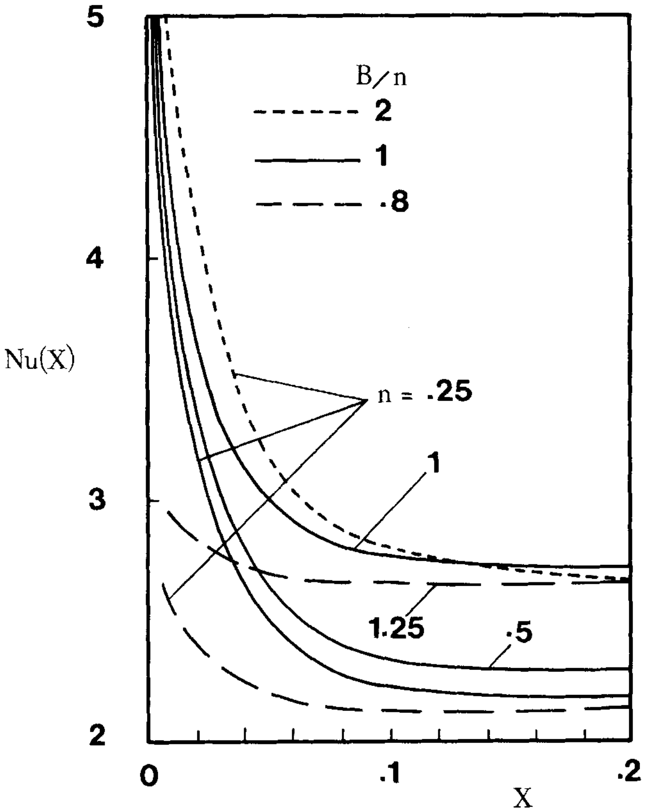

Fig. 7. Variation of the local stationary plate Nusselt number for $\Theta_i = 1$, and $(Pe.Ec/Re) = 1$.

Table 1. Eigenvalues for Constant Plates Temperatures

B/n	0.3	0.5	0.8	0.9	1.0	2.0
λ_1	39.25	18.88	9.86	8.43	7.35	3.36
λ_2		39.22	20.55	17.58	15.34	7.35
λ_3			31.25	26.74	23.34	11.34
λ_4			41.94	35.90	31.34	15.34
λ_5				45.05	39.34	19.34
λ_6					47.33	23.25
λ_7						27.34
λ_8						35.24
λ_9						39.24
λ_{10}						43.24
λ_{11}						47.24

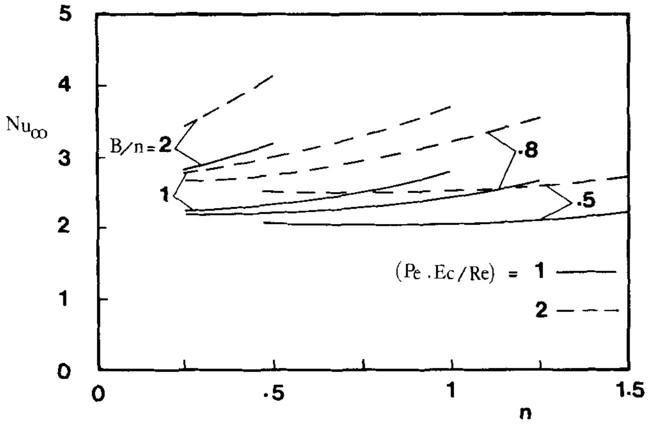

Fig. 8. Variation of the asymptotic stationary plate Nusselt number with the flow behavior index for $\theta_i = 1$.

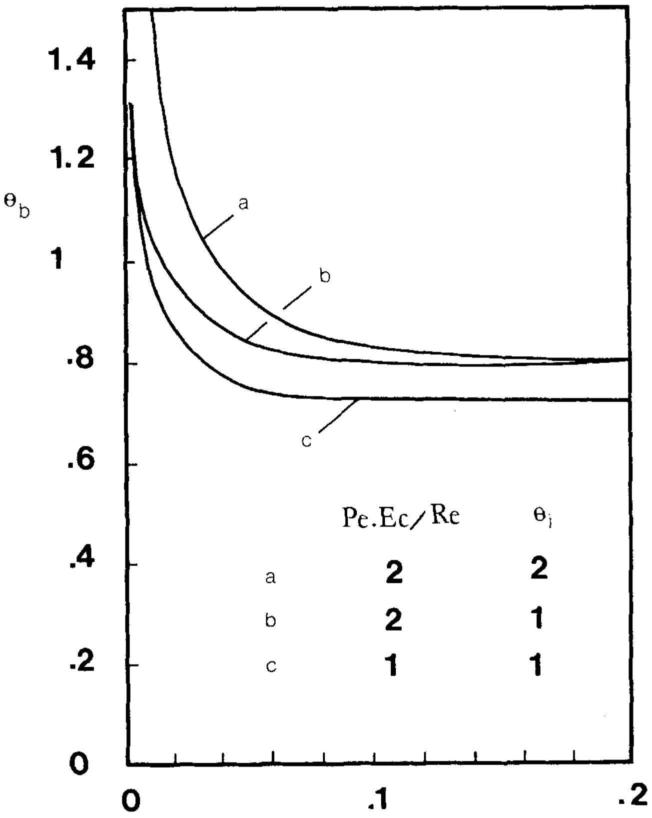

Fig. 9. Variation of the cup-mixing or bulk temperature for $B/n = 1$, and $n = 0.25$.

$$Nu(x) = h(x)a/k = -a[(\partial T/\partial y)_{y=0}]/(T_0 - T_b) \tag{20}$$

which can be written as

$$Nu(X) = [(\partial\theta/\partial Z)_{Z=1}]/\theta_b \tag{21}$$

where θ_b is the bulk or cup mixing temperature and is given by

$$\theta_b = (T_b - T_0)/(T_1 - T_0) = \int_1^H u^* \theta \, dZ \Big/ \int_1^H u^* \, dZ \tag{22}$$

RESULTS AND DISCUSSION

The magnitude of the parameter B as given by Equation (6) is limited to the range $0 < B < 1$. Only when $n = 1$, in general, are other values outside this range possible. But then these would correspond to the case of positive pressure gradients, which is not the concern in this work. It is also noticed from Figure 2 that the magnitude of B is nearly the same for all values of n when $(-Re \cdot dp^*/dx^*) < 1$. Therefore, one can approximate this range by

$$B = 2/[1 + 2/(-Re \cdot dp^*/dx^*)] \tag{23}$$

When Equation (13) is used to calculate the velocity profile, the maximum relative deviation from the exact profile, as given by Equation (5), is found to be less than 3% for B less than 0.75. The deviation then rises more rapidly for B greater than 0.75 but does not exceed 9% at the maximum value of $B = 1$. This is shown in Figure 4. One then expects reasonably good results by using the approximate velocity profile to solve for the temperature. In the limit when $B \to 0$, Equation (10) gives the solution already reported by Tien (1961).

Table 1 gives the eigenvalues as calculated from Equation (19) which is valid only for constant plates temperatures. The calculation was done for values of $(B/n) = 0.1$ up to 2.0, and the computations were stopped whenever λ exceeded 50. As shown in Table 1, the trend indicates that for a given pressure gradient, the larger the value of n, the larger the magnitude of the first eigenvalue, and thus the shorter the entrance length. In other words, the dilatant fluids would attain the developed temperature profiles faster than the pseudoplastic fluids. In general, the solution for the temperature shows that its magnitude depends mainly on (B/n) rather than on B alone. Figures 5 and 6 show the temperature distribution between the plates for constant plates temperatures. Figure 7 shows that the local Nusselt number $Nu(X)$ is strongly dependent on (B/n) at a fixed value of n. Figure 8 shows the dependence of the asymptotic Nusselt number Nu_∞ on (B/n) and n. Figure 9 shows the variation of the bulk or cup mixing temperature with the longitudinal distance X. It also shows that the effect of the initial temperature, at $X = 0$, diminishes rapidly, while that of the parameter $(Pe \cdot Ec/Re)$ is not important for small values of X.

When either $n = \infty$ or $B = 0$, the velocity profile becomes linear; that is, $u^* = y^*$. In this case, the solution for θ_1 becomes essentially that given by Bruin (1972) for the case of zero pressure gradient, or that given by Sestak and Rieger (1969).

NOTATION

a = distance between plates, m
c_p = specific heat of fluid, J/kg °K
k = thermal conductivity of fluid, J/s m °K

p = pressure of the fluid, N/m^2
T = temperature of the fluid, °K
u = velocity component in x direction, m/s
U = velocity of upper plate, m/s
x = streamwise coordinate, m
y = transverse coordinate, m

Greek Letters

λ = eigenvalue
κ = fluid consistency, N s^n/m^2
ρ = fluid density, kg/m^3
θ = dimensionless temperature, $(T - T_0)/(T_1 - T_0)$
τ = shear stress in fluid, N/m^2

Dimensionless Groups

Ec = Eckert number, $U^2/c_p(T_1 - T_0)$
M = Kummer's function
n = flow behavior index
Nu = Nusselt number, $-a(\partial T/\partial y)_0/(T_0 - T_b)$
Pe = Peclet number, $\rho c_p U\ a/k$
Re = Reynolds number, $\rho\ U^{2-n}\ a^n/\kappa$

Subscripts

0 = lower (stationary) plate
1 = upper (moving) plate
i = entrance at $x = 0$

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Second Virial Coefficients of Water Pollutants

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The major pollutants other than oil in petroleum refinery wastewater streams were identified in a recent paper on ionization constants (Tsonopoulos et al., 1976). They are the following electrolytes: ammonia, hydrogen sulfide, phenols, chlorides, cyanides and thiocyanates, and alkyl mercaptans. With the exception of the chlorides and thiocyanates, these pollutants are volatile and can therefore be removed from the wastewater, to a lesser or greater extent, by vaporization. To predict the vaporization of the volatile pollutants we must know, in addition to their ionization constants and Henry's constants in water, their fugacity coefficients. These fugacity coefficients can be calculated if, in turn, the second virial coefficients are known. This paper presents the second virial coefficients of the volatile water pollutants and their mixtures in terms of an empirical correlation (Tsonopoulos, 1974).

Fugacity coefficients correct for the deviation from vapor phase ideality and are therefore needed in any vapor-liquid equilibrium calculations, especially those at superatmospheric pressures. As long as the density of the vapor mixture is not greater than one fourth its critical value, the following equation can be used to determine the value of ϕ_i, the fugacity coefficient of component i (in a binary mixture):

$$\ln \phi_i = \frac{2}{v_M}\ (y_i\, B_{ii} + y_j\, B_{ij}) - \ln z_M \tag{1}$$

v_M and z_M are the molar volume and the compressibility factor of the vapor mixture, and y_i is the vapor mole fraction of component i. B_{ii} is the second virial coefficient of pure i, and B_{ij} is the second virial cross coefficient. Since

$$z = Pv/RT = 1 + B/v \tag{2}$$

ϕ_i can be determined once B is known. This, in turn, can be obtained from experimental data or calculated with a correlation. The latter approach is taken here.

Complete information on the equations will be found in the 1974 paper. Similarly, the references given in 1974 will not be repeated here.

PURE COMPOUNDS

The B correlation involves three terms that are functions of the reduced temperature:

$$\frac{BP_c}{RT_c} = f^{(0)}(T_R) + \omega f^{(1)}(T_R) + f^{(2)}(T_R) \tag{3}$$

$f^{(2)}$, which is the polar contribution to B, involves two parameters, although of the components considered here only water requires both of them (owing to hydrogen bonding):

$$f^{(2)}(T_R) = \frac{a}{T_R{}^6} - \frac{b}{T_R{}^8} \tag{4}$$

a depends strongly on μ_R, the reduced dipole moment, but not through a universally applicable relationship. The following relationship applies to ketones, aldehydes, alkyl nitriles, and ethers (Tsonopoulos, 1974):

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